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LETTER TO THE EDITOR

Uniaxial compression effects on 2D mixtures of 'hard' and 'soft' cylinders

T Travers, D Bideau, A Gervois†, J P Troadec and J C Messager

Groupe de Physique Cristalline, UA CNRS 040804, UER SPM, Université de Rennes I, 35042 Rennes Cedex, France

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Abstract. An experimental study of uniaxial compression of a mixture of 'hard' and 'soft' cylinders is made. The macroscopic strain-stress law strongly depends on the geometrical and compositional heterogeneities. Two changes in the behaviour are observed: one is weak, at the percolation threshold, and the other is strong, at the 'rigidity' threshold.

The mechanical behaviour of dense granular media is extremely complex, even in the simple case of the strain under weak uniaxial pressure. Indeed, this strain depends on the medium structure, the nature of the contacts between grains and their geometry, eventually the wall effects, and also on the amplitude of the applied pressure: these systems generally have a non-linear strain-stress characteristic, as we shall see later.

Therefore we are led to study simplified experimental models. Some authors (Dantu 1957, de Josselin de Jong and Verriujt 1969, Faugeras and Gourves 1980) have already underlined the interest of a two-dimensional model called the Schneebeli model (Schneebeli 1956), which is a two-dimensional (2D) packing of cylinders with parallel axes. This model is of great interest for several reasons.

(i) 'Grains' have a simple geometrical shape.

(ii) It is relatively easy to determine the geometry of the packing from direct observation or photograph. The geometry is difficult to study in three-dimensional (3D) experiments with spheres and spheroidal objects.

(iii) Photoelastic experiments can give some information on the space repartition of the intergranular stresses (at a 'microscopic' level) and then allow better understanding, at least phenomenologically, of the effects of a macroscopic stress on the system. This is illustrated by a very beautiful film by Dantu‡.

Most of the mechanical and photoelastic studies on this model were carried out on packings of cylinders with the same physical properties, but with a grain size distribution (Dantu 1967, Drescher and de Josselin de Jong 1972): in this case, the medium presents a 'strong' geometrical disorder (Rubinstein and Nelson 1982), as in real media. For a binary distribution in particular, photoelastic studies have shown that this distribution was not mechanically neutral: the paths of the largest stresses preferentially pass through the large grains (Oger *et al* 1986).

Nevertheless, even in the case of regular packings of equal size cylinders, Dantu (1957) showed, with photoelastic experiments, that the repartition of the stresses is inhomogeneous: the strongest stresses draw up a network ignoring a large part of the

† Permanent address: Service de Physique Théorique, CEN Saclay, 91191 Gif-sur-Yvette Cedex, France.
‡ This film is presently available at the Laboratoire Central des Ponts et Chaussées, Paris.

grains in the system. In the case of uniaxial compression, links follow the principal stress direction. This heterogeneity in the stress distribution is, in fact, created by weak geometrical heterogeneities—for instance, weak differences in the cylinder diameters. It is of great importance in the case of weak strains as here.

In many granular media geometrical heterogeneities and heterogeneities arising from composition (grains with different physical properties) coexist. Thus, mixtures of conducting and insulating spheres are good structural models for percolation (Troadec and Bideau 1981). For us, it is interesting to study, on a Schneebeli model, the effects of a 'mechanical' contrast on mixtures of 'hard' and 'soft' grains. This letter is a first experimental approach to this problem, in the particular case of equal size cylinders in regular triangular packings.

The mixtures studied are composed of plexiglass (proportion p) and rubber (proportion $1-p$) cylinders. The ratio between bulk Young's moduli is ~ 2000. These materials have been chosen for two reasons: the 'soft' material must not be too soft, in order to avoid initial strains under gravity, and the plexiglass has the advantage of having a good photoelastic response.

The length (2.5 cm) of the rods has been chosen to avoid a global buckling of the system under pressure. But there are fluctuations in the diameter of the plexiglass elements and their sections along the axis are not really circular; these defects are in part due to the annealing necessary to eliminate the residual stresses prejudicial to photoelastic studies. One can consider that their diameter is 4 ± 0.1 mm. The variations of the rubber 'grain' diameter around the same mean value are very small. The packings are constituted by 48 horizontal rows, with alternately 44 and 45 sites (2136 'grains'). The size and the shape of the sample have been chosen to avoid wall effects, which can greatly modify the macroscopic deformation quantitatively and qualitatively (Dantu 1967). These wall effects are essentially due to the friction between the grains and the wall of the container, and can lead to an important gradient in the spatial distribution of the constraints in the system along the pressure axis. In our case, several tests with different frictions at the wall show that these effects are unimportant. The system is put in a rigid frame and placed in an Instron 1175 universal testing machine. A vertical displacement Δh, at a given speed, is imposed on an upper plate and we measure the corresponding compressive force F.

The experiments are made at low speed (0.5 mm min^{-1}): the observed phenomena are then speed independent. The force F varies in the range 0-2000 N. For the pure rubber samples, all the cycles are reproducible; for the compression, in the whole range studied, a power law is observed (figure 1):

$$F = F_0(\Delta h/h)^m \tag{1}$$

with $m \sim 1.4 \pm 0.1$ and $F_0 \sim 4 \times 10^4$ N. h is the initial height of the sample. For plexiglass, during the first application of the pressure, the first part of the compression is 'plastic', i.e. there are important deformations of the structure by grain sliding; following pressure cycles are reproducible and then we have, for the compression, the same law (1) with $m \sim 3.9 \pm 0.3$ and $F_0 \sim 10^{11}$ N (figure 1).

We observe such a power-law dependence for all the mixtures studied (any value of p between 0 and 1) and we examine the variations of the two quantities m and F_0 (in fact $\ln F_0$, because of the great amplitude of the variations) with the composition p.

Figures 2 and 3 summarise our experimental study. The values m and $\ln F_0$ are averages obtained from independent measurements on at least three different samples

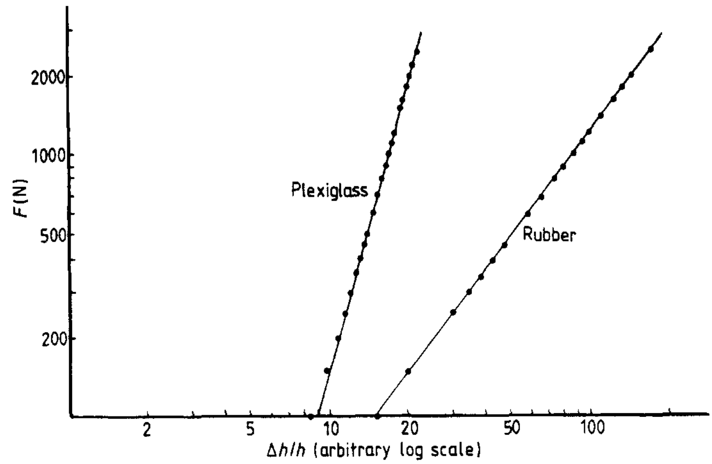

Figure 1. Measured compressive force F as a function of the imposed vertical displacement Δh for $p=0$ and $p=1$. At $F=2000$ N, $\Delta h/h \sim 10\%$ for rubber, 1% for plexiglass.

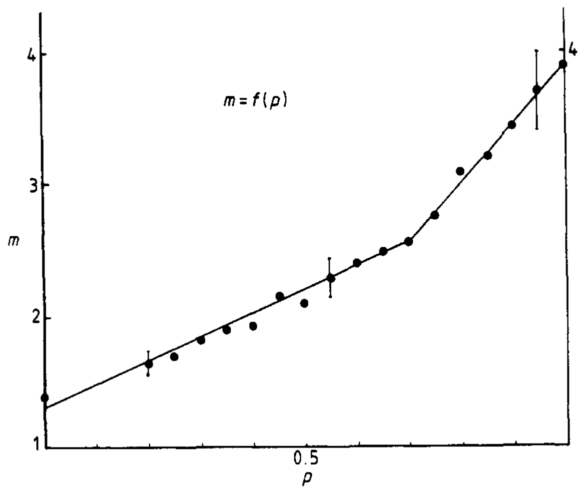

Figure 2. Variations with p of the macroscopic exponent m of the experimental law (1): $F=F_0(\Delta h/h)^m$.

with the same composition p (10 for $p=1$). The curve $\ln F_0 = f(p)$ shows a weak fracture at $p=0.5$ and a stronger one at $p=0.7$. The value $p=0.5$ corresponds to the percolation threshold p_c of the regular triangular site problem. The variations of $m(p)$ also show a fracture at $p=0.7$; the accuracy of the measurements does not allow us to decide whether or not there is a behaviour change at $p=0.5$.

The first problem is the justification of the experimental power law (1). At the microscopic level, the stress transmission in a granular material is localised at the contact between grains where most of the deformation takes place. In the case of spherical grains, the stress f applied to a contact, assumed to be radial, is related to the decrease Δd of the distance between the grain centres by the Hertz law

$$f = g(E_1, E_2, R_1, R_2)(\Delta d)^x \quad (2)$$

where E_i and R_i are the bulk Young's modulus and the radius of the grain i, respectively,

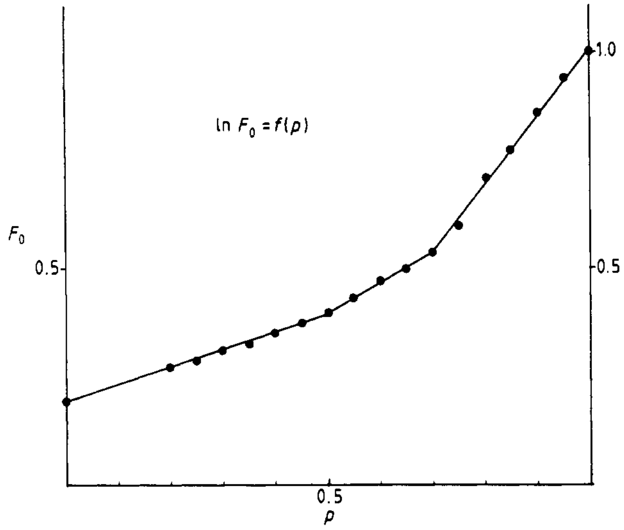

Figure 3. Variation with p of ln F_0, where F_0 is the prefactor of the experimental law (1).

and $x = 1.5$. In the case of cylinders, it is not realistic to consider that the contact is perfect all along the axis: the cylinders are weakly crossed. In this case, we again have a law like (2) with the same exponent $x = 1.5$ as for spheres (Landau and Lifshitz 1959). In fact, the Hertz law does not take into account the microscopic roughness of the grain surface nor the possible tangential component of the stress. Nevertheless, these two defects will not lead to sensitive variations of the value of the exponent x (Mindlin 1954, Georges 1986). Experimentally, for two plexiglass half-cylinders in contact, and for a range of stresses larger than that between two grains in our samples, a Hertz law is found with $x = 1.8 \pm 0.1$, in agreement with the above provisions.

According to Hooke's theory, at the macroscopic level a homogeneous solid submitted to a weak and slow uniaxial compression has an axial strain $\varepsilon_a = \Delta h/h$ related to the stress σ_a by

$$\varepsilon_a = \frac{1 - 2\nu K_0}{E_v} \sigma_a \tag{3}$$

where E_v is the Young's modulus in the direction of the compression, ν is the Poisson ratio and K_0 is the coefficient of lateral pressure; these terms are supposed to be constant. Generally, the granular materials do not have such a linear response, because of the complexity of the deformation mechanisms: the 'Young's modulus' depends on the macroscopic stress applied to the system. One can admit an incremental form of (3), $\mathrm{d}\sigma_a = E' \, \mathrm{d}\varepsilon_a$, where E' is the uniaxial modulus of deformation. With the hypothesis of a power law between E' and σ_a (hypoelasticity in the sense of Truesdell), we have a macroscopic law (Feda 1982):

$$\varepsilon_a = \frac{1}{E_0(1 - m')} \left(\frac{\sigma_0}{\sigma_a} \right)^{m'} \tag{4}$$

with $0 < m' \leqslant 1$.

The experimental law (1) and the law (4) have the same form, with $m' = (m - 1)/m$. However, it is difficult to relate the macroscopic parameters to the different mechanisms of the deformation in the two cases.

In our mixtures, the deformation depends on several factors whose relative importance varies with the composition. Our results could be interpreted in terms of mechanical percolation. However, because of the small size of our system, we only study the non-critical aspects of the problem.

The mechanical behaviour of the samples appears to be different above and below $p \sim 0.7$. This value corresponds to a rigidity threshold p_R, i.e. the composition above which the 'hard' infinite cluster is stable against the macroscopic force. This threshold p_R is comparable to $p_{cen} = 0.65 \pm 0.005$ obtained by Lemieux *et al* (1985) on triangular elastic networks with central forces; in particular, the triangles formed by three plexiglass grains in contact play an important role in the stable structure. Between the percolation and the rigidity thresholds ($p_c < p < p_R$) the connected cluster of hard grains is not stable against forces, as we have verified by photoelastic experiments. The sample is placed between crossed polarisers. The stressed grains are then bright and the unstressed grains are dark. The division between stressed and unstressed grains is in fact Manichaean: only grains on which the constraints are larger than some value σ^* appear bright. The number of bright grains then varies with the applied force and with the quality of the optical apparatus. Figure 4 shows a sample on which a macroscopic force $F = 2000$ N is applied. Above the threshold p_R, the system presents a continuous network of bright links as described by Dantu (1957). Below p_R we do not observe such a network, even if some finite lines of bright cylinders can be seen locally.

Below p_R, the system can be structurally described as formed by finite clusters ($p < p_c$) and finite clusters plus one infinite cluster ($p > p_c$) of hard cylinders. These clusters are immersed in an elastic medium and all their components are relevant for the elastic properties of the system. So, for example, as seen in photoelasticity

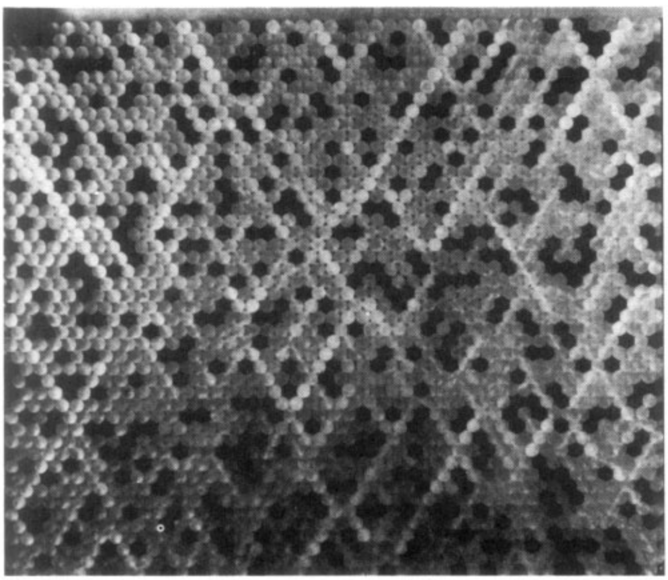

Figure 4. Photograph of a sample between crossed polarisers for $p = 0.8$. The 'constrained' grains appear light and the 'non-constrained' ones are dark (a macroscopic stress $F = 2000$ N is applied to the sample).

experiments, dead ends can dissipate some energy by contraction and bending. Part of the infinite cluster made up of rigid regions of hard cylinders has more and more importance in the transmission of the forces when p approaches p_R. The change in the behaviour of F_0 and eventually of m at p_c could be attributed to the fact that the more important contribution to the energy comes from the bending of the links of the infinite cluster when this latter cluster appears (Kantor and Webman 1984).

Above p_R, the local inhomogeneities in the stable structure formed by plexiglass cylinders due to geometrical imperfections (deviation from a cylinder form, grain size distribution, etc) are of the same order as, or larger than, the local deformations (Hertz deformations). Beside composition heterogeneity, this is a new factor that must be taken into account in the variations of m and F_0. In particular, for $p=1$, where we have only geometrical defects, these lead to an inhomogeneous spatial distribution of the intergranular stresses and the macroscopic deformation depends on large scale mechanisms; the macroscopic exponent ($m = 3.9 \pm 0.3$) is different from the microscopic one ($m = 1.8 \pm 0.1$ measured on two half-cylinders of plexiglass) and presents larger fluctuations from sample to sample (this is in contrast with the exponent 1.4 ± 0.1 at $p=0$ which is compatible with the Hertz exponent). Thus, experimental observations seem to show that geometrical defects are dominant above p_R.

The experiment described above is the first step of a more exhaustive study. Many points have still to be clarified. Presently, transmission of the forces above the threshold is analysed in special arrangements of soft and hard cylinders (to be published) and we are beginning experiments with 'very hard' grains to obtain a better contrast between phases (mixtures of steel and rubber cylinders).

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